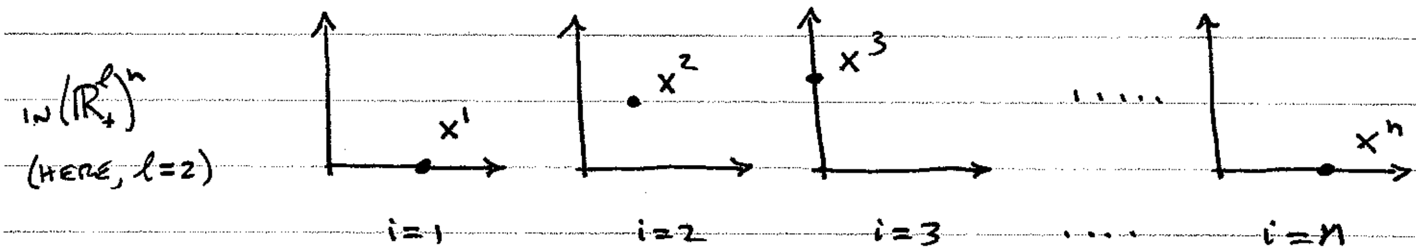
EFFICIENCY PRICES
(DECENTRALIZING AN EFFICIENT ALLOCATION)

CONSIDER CONSUMERS $i = 1, \dots, n \in N$ AND GOODS $k = 1, 2$. ASSUME THAT EACH u^i IS SELFISH, INCREASING, QUASICONCAVE, AND CONTINUOUSLY DIFFERENTIABLE.

CONSIDER AN ALLOCATION $(x^i)_1^n = (x^1, x^2, \dots, x^n)$:



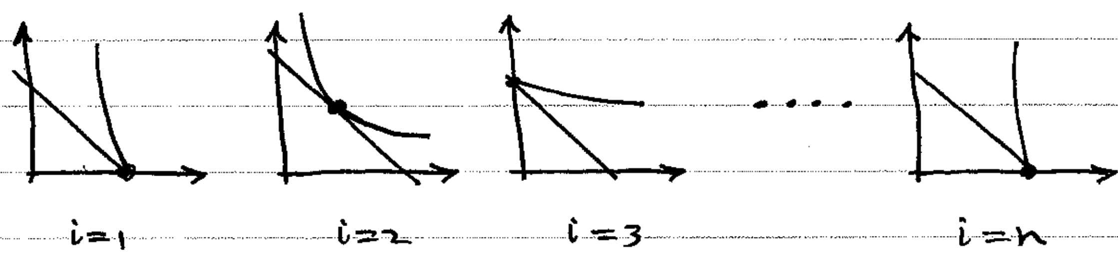
SUPPOSE $(x^i)_1^n$ IS PARETO EFFICIENT. THEN WE HAVE NUMBERS $\lambda_1, \lambda_2, \dots, \lambda_n \geqq 0$ AND $\sigma_1, \sigma_2 > 0$ THAT SATISFY

$$\text{(FOC)} \quad \left\{ \begin{array}{ll} \lambda_i u_1^i \leqq \sigma_1 , & \text{w/ EQ. IF } x_1^i > 0 \\ \lambda_i u_2^i \leqq \sigma_2 , & \text{w/ EQ. IF } x_2^i > 0. \end{array} \right\} \forall i$$

LET $s = \dfrac{\sigma_1}{\sigma_2}$. THEN WE HAVE

$$(*) \quad \left\{ \begin{array}{ll} MRS^i \geqq s & \text{IF } x_1^i > 0 \\ MRS^i \leqq s & \text{IF } x_2^i > 0 \end{array} \right\} \forall i.$$

DIAGRAMATICALLY, (*) TELLS US THAT FOR ~~EACH~~ $i \in N$ WE CAN DRAW A LINE THROUGH x^i WITH SLOPE $-s$, AND THE I-CURVE THROUGH x^i WILL LIE ABOVE THAT LINE: IN OTHER WORDS, THE "SHADOW VALUES" σ_1 AND σ_2 GENERATED BY A PARETO EFFICIENT ALLOCATION ARE MUCH LIKE EQUILIBRIUM



PRICES. IN FACT, THEY ARE CALLED "EFFICIENCY PRICES" OR "DECENTRALIZING PRICES."

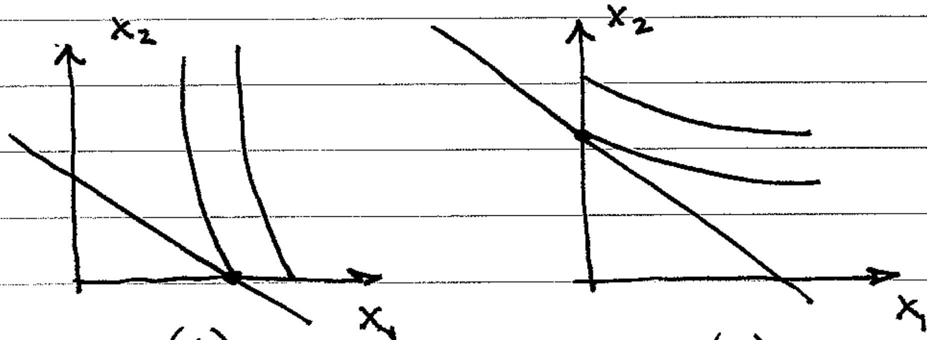
CONVERSELY, IF $(x^i)_1^n$ IS AN ALLOCATION FOR WHICH SUCH A NUMBER s EXISTS (AND FOR WHICH $\sum_{i=1}^{n} x^i = \overset{\circ}{x}$), THEN $(x^i)_1^n$ MUST BE PARETO EFFICIENT.

EFFICIENCY PRICES

At a Pareto allocation $\exists\, \sigma_1, \sigma_2 > 0$ s.t.
($\forall i \in N$)

(A) If $x_1^i > 0$, then $MRS^i \geq \frac{\sigma_1}{\sigma_2}$.

(B) If $x_2^i > 0$, then $MRS^i \leq \frac{\sigma_1}{\sigma_2}$.



(A) (B)

There is a slope $s = \frac{\sigma_1}{\sigma_2}$ s.t.

<u>For every $i \in N$</u>: (A) and (B) hold.

We call σ_1 and σ_2 <u>efficiency prices</u> or <u>decentralizing prices</u> for this Pareto allocation; s would be the efficiency relative price.

An allocation is Pareto efficient if and only if $\exists\, \sigma_1, \sigma_2 > 0$ s.t. (A) and (B) are true for every $i \in N$.